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DESIGN OF LINEAR CONTINUOUS MULTIVARIABLE PROPORTIONAL-PLUS-INTEGRAL-PLUS-DERIVATIVE CONTROLLER

Ahmed I.A. Salama

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DESIGN OF LINEAR CONTINUOUS MULTIVARIABLE PROPORTIONAL-PLUS-INTEGRAL-PLUS-DERIVATIVE CONTROLLER

by

Ahmed I.A. Salama*

ABSTRACT

A time-domain design technique of proportional-plus-integral-plus-derivative controllers for linear continuous multivariable systems is presented. The proposed design approach utilizes a null space formulation of the eigenstructure problem. The plant output is assumed to be available for measurement. The resulting controller assigns a prescribed set of self-conjugate eigenvalues and a corresponding set of admissible self-conjugate generalized eigenvectors to the closed-loop system matrix. The proposed technique is flexible and straightforward compared to known techniques.

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* Research Scientist, Process Control and Computer Applications Section, Coal Research Laboratory, Edmonton, Alberta, CANMET Energy, Mines and Resources Canada

CONCEPTION D'UN RÉGULATEUR NUMÉRIQUE PROPORTIONNEL PAR INTÉGRATION/ PAR DÉRIVATION POUR SYSTÈME MULTIVARIABLE CONTINU LINÉAIRE

par

Ahmed I.A. Salama*

RÉSUMÉ

Une technique de conception à domaine temps de régulateurs numériques proportionnels par intégration/par dérivation pour systèmes multivariables continus linéaires est présentée. La conception proposée se fonde sur la formulation dans l'espace zéro aux valeurs propres. On présume que la sortie du système peut être mesurée. Le régulateur assigne un ensemble prescrit de valeurs propres autoconjuguées et un ensemble correspondant de vecteurs propres généralisés autoconjugués admissibles à la matrice système à boucle fermée. La technique proposée est souple et se compare directement aux techniques connues.

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*Chercheur scientifique, Section du contrôle des procédés et des applications sur ordinateur, Laboratoire de recherche sur le charbon, Edmonton (Alberta), CANMET, Énergie, Mines et Ressources Canada.

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INTRODUCTION

Recently there has been much interest in the problem of designing proportional-plus-integral-plus-derivative (PID) controllers for linear continuous multivariable systems. This type of controller utilizes two extra actions i.e. integral of error and derivative of output compared to the proportional output feedback controller, to achieve acceptable performance in regard to static accuracy, disturbance rejection and transient response.

Most of the PID controller design techniques assume either that both system state and output are available for measurement **(1-5)** or that system output only is available for measurement **(6,7)** . The latter approach is superior to the former because in many situations some of the system states are not available for measurement. The proposed design techniques assume the system is under control (controllable) in state space formulation and subsequently determine the PID controller feedback gain matrices either in the time-domain **(1-6)** or in the frequency-domain **(7)** . The time-domain approach has advantages over the frequency-domain approach. It avoids the computation of the system transfer function matrix from its state variable formulation which can be highly inaccurate **(8)** . Some authors assume structural constraint on the feedback gain matrices of the PID controller in their design techniques, namely, that of unit rank **(3,7,9)** . In this case the degrees of freedom in the controller parameters are greatly reduced. The Lypanuv technique is utilized in designing a state feedback PID controller **(4,5)** . A procedure for designing a PID controller for a multivariable system to track a polynomial type input has also been reported **(9)** .

The main feature of most of the PID controller design techniques is that they mainly address the eigenvalue-assignment problem **(1-7,9)** . However, the problem of eigenvalue-eigenvector assignment has practical significance in regard to modal control and consequently it will be the subject of the present report.

In this report, a time-domain approach for designing PID controllers for linear continuous time-invariant systems is adopted. The system output is assumed to be available for measurement. The eigenstructure

formulation is utilized to determine the controller feedback gain matrices. The resulting matrices assign a prescribed set of self-conjugate eigenvalues and a corresponding set of admissible self-conjugate generalized eigenvectors to the closed-loop system matrix. The proposed PID controller design technique has the following advantages: i) it involves only output feedback thus it covers most practical situations, ii) it is a time-domain approach, thus there is no computation of the transfer function matrix of the closed-loop system from its state variable formulation, iii) it imposes no structural constraint on the output feedback matrices, thus it allows greater freedom in the design, and iv) it assigns prescribed eigenvalues and corresponding admissible generalized eigenvectors to the closed-loop system matrix both of which are required for modal control. The problem investigated in this report can be considered an extension to the problem of eigenvalue-generalized eigenvector assignment using proportional-plus-integral controller (10) .

PROBLEM STATEMENT

Consider the linear continuous time-invariant multivariable system described in state space form as follows

$$\dot{X} = AX + BU$$
$$Y = CX \qquad \text{Eq 1}$$

where X is the n x 1 state vector, U is the m x 1 input vector, Y is the q x 1 output vector and A,B,C are constant matrices of appropriate dimensions. The output feedback law is assumed to be of the form.

$$U = G_P Y + G_D \dot{Y} + G_I \int_0^t (Y - Y_d)dt, \qquad \text{Eq 2}$$

where G_P, G_I and G_D are constant m x q proportional, integral and derivative output feedback gain matrices respectively (Fig. 1). Substitution of the output equation Y = CX in Eq 2 yields

$$U = G_P CX + G_D C\dot{X} + G_I C \int_0^t X dt + \Theta \qquad \text{Eq 3}$$

where,

$$\Theta \overset{\Delta}{=} G_I \int_0^t Y_d dt.$$

Substitution of Eq 3 in Eq 1 yields the closed-loop system

$$\dot{X} = (I - BG_D C)^{-1} \left((A + BG_P C)X + BG_I C \int_0^t X dt + B\Theta\right), \qquad \text{Eq 4}$$

providing that $(I - BG_D C)$ is invertible. Now let us introduce the variable Z as

$$Z = \int_0^t Y dt, \qquad \text{Eq 5}$$

then Eqs 4 and 5 can be written in state variable form as

$$\dot{X}_c = A_c X_c + B_c \theta$$
$$Y = C_c X_c \qquad \text{Eq 6}$$

where,

$$A_c = \begin{bmatrix} (I-BG_D C)^{-1}(A+BG_P C) & (I-BG_D C)^{-1}BG_I \\ C & 0 \end{bmatrix}, \qquad \text{Eq 7}$$

$$X_c = \begin{bmatrix} X \\ Z \end{bmatrix}, \qquad \text{Eq 8}$$

$$B_c = \begin{bmatrix} (I-BG_D C)^{-1}B \\ 0 \end{bmatrix}, \quad C_c = [C \;\; 0] \; . \qquad \text{Eq 9}$$

The dimensions of X_c, A_c, B_c and C_c are $(n+q)$, $(n+q)x(n+q)$, $(n+q)xm$ and $qx(n+q)$ respectively.

The problem of eigenvalue-generalized eigenvector assignment of the system represented by Eq 1 via a proportional, integral and derivative output feedback law of the form of Eq 3 (where the integrator state is governed by Eq 5) is one of determining the real matrices G_P, G_I and G_D such that the closed-loop matrix A_c has a prescribed set of self-conjugate eigenvalues and a corresponding set of admissible self-conjugate generalized eigenvectors.

EIGENVALUE-GENERALIZED EIGENVECTOR STRUCTURE

EIGENVALUE ≠ ZERO

Consider a particular set of self-conjugate eigenvalues of A_c, $\lambda_i \neq 0$, $i=1,2,\ldots\ldots,l$, $l \leq (n+q)$. Let the eigenvalue λ_i has a geometric multiplicity k_i and an algebraic multiplicity m_i. The generalized eigenvectors $U^i_{\mu,\rho}$'s associated with λ_i must satisfy

$$[A_c - \lambda_i I]U^{i,\mu,\rho} = U^{i,\mu,\rho-1} \qquad \text{Eq 10}$$

where,

$$\rho = 1,2,\ldots,m_{i,\mu}, \quad \mu = 1,2,\ldots,k_i,$$
$$i = 1,2,\ldots,\ell,$$

with $U^{i,\mu,0}=0$, also $m_{i,\mu}$ must satisfy

$$m_i = \sum_{\mu=1}^{k_i} m_{i,\mu}. \qquad \text{Eq 11}$$

The dimension of A_c is related to m_i as

$$(n+q) = \sum_{i=1}^{\ell} m_i . \qquad \text{Eq 12}$$

Let the generalized eigenvector $U^{i,\mu,\rho}$ be partitioned as

$$U^{i,\mu,\rho} = ((U_1^{i,\mu,\rho})', (U^{i,\mu,\rho})')', \qquad \text{Eq 13}$$

where the ' denotes vector transpose. The dimensions of the subvectors $U_1^{i,\mu,\rho}$ and $U^{i,\mu,\rho}$ are n x 1 and q x 1 respectively. Using Eqs 7 and 10 we get

$$((I-BG_DC)^{-1}(A+BG_\rho C)-\lambda_i I)U_1^{i,\mu,\rho}+(I-BG_DC)^{-1}BG_I U^{i,\mu,\rho}=U_1^{i,\mu,\rho-1} \qquad \text{Eq 14}$$

$$CU_1^{i,\mu,\rho} - \lambda_i U^{i,\mu,\rho} = U^{i,\mu,\rho-1} \qquad \text{Eq 15}$$

Using Eq 15 we get

$$v^{i,\mu,\rho} = (CU_1^{i\,\mu\,\rho} - v^{i,\mu,\rho-1})/\lambda_i \quad . \qquad \text{Eq 16}$$

From Eq 16 it is obvious that $v^{i\,\mu\,\rho}$ can be expressed in terms of $U_1^{i\,\mu\,\rho}$. Substituting Eq 16 into Eq 14 yields

$$(A-\lambda_i I)U_1^{i\,\mu\,\rho} + B[G_p CU_1^{i\,\mu\,\rho} + G_I(CU_1^{i\,\mu\,\rho} - v^{i\,\mu\,\rho-1})/\lambda_i +$$

$$+ G_D(\lambda_i CU_1^{i\,\mu\,\rho} + CU_1^{i\,\mu\,\rho-1})] = U_1^{i\,\mu\,\rho-1} \qquad \text{Eq 17}$$

or in matrix form as

$$[A-\lambda_i I, B]\begin{bmatrix} U_1^{i\,\mu\,\rho} \\ W^{i\,\mu\,\rho} \end{bmatrix} = U_1^{i\,\mu\,\rho-1} \qquad \text{Eq 18}$$

where,

$$\rho = 1,2,\ldots\ldots,m_{i\,\mu}, \qquad \mu = 1,2,\ldots\ldots,k_i$$
$$i = 1,2,\ldots\ldots,\ell$$

and

$$W^{i,\mu,\rho} = G_p CU_1^{i,\mu,\rho} + G_I(CU_1^{i\,\mu\,\rho} - v^{i\,\mu\,\rho-1})/\lambda_i +$$

$$+ G_D(\lambda_i CU_1^{i,\mu,\rho} + CU_1^{i,\mu,\rho-1}) \quad . \qquad \text{Eq 19}$$

Let the eigenvalues λ_i $(i=1,2,\ldots\ldots,\ell)$ form a complete set of self-conjugate eigenvalues, and choose the integers $m_{i\,\mu}$ and k_i so that the entire set of vectors $U^{i\,\mu\,\rho}$'s not only satisfies Eq 10, but is also linearly independent and self-conjugate. The real output feedback gain matrices G_p, G_I and G_D can then be expressed using Eqs 18 and 19:

$$[G_p, G_I, G_D]\begin{bmatrix} \cdots\cdots, & CU_1^{i\,\mu\,\rho} & ,\cdots\cdots \\ \cdots\cdots,(CU_1^{i\,\mu\,\rho} - U^{i\,\mu\,\rho-1})\Lambda_i & & ,\cdots\cdots \\ \cdots\cdots,(\lambda_i CU_1^{i\,\mu\,\rho} + CU_1^{i\,\mu\,\rho-1}) & & ,\cdots\cdots \end{bmatrix}$$

$$= [\cdots\cdots, W^{i\,\mu\,\rho}, \cdots\cdots] \qquad \text{Eq 20}$$

where,

$$\rho = 1,2,\cdots\cdots,m_{i\,\mu}, \quad \mu = 1,2,\cdots\cdots,k_i,$$
$$i = 1,2,\cdots\cdots,\ell.$$

Expressed in compact form:

$$[G_p, G_I, G_D]U = W, \qquad \text{Eq 21}$$

where the matrix U has dimension $3q \times (n+q)$. If the matrix U is invertible then we can determine G_p, G_I and G_D from Eq 21 as follows

$$[G_p, G_I, G_D] = WU^{-1}. \qquad \text{Eq 22}$$

Three cases are to be investigated, namely, $(n+q)=3q$, $(n+q)<3q$ and $(n+q)>3q$. In the case $(n+q)=3q$ or $n=2q$, Eq 22 yields a unique solution for G_p, G_I and G_D, if it exists. However if $(n+q)<3q$ or $n<2q$, then we are required to impose some constraints on the elements of G_p, G_I and G_D such as setting some elements, and/or the sum of some elements, and/or the row sum of some rows equal to zero. This can be achieved by introducing suitable $(2q-n)$ columns in W. Then employing Eq 22, we can determine G_p, G_I and G_D. For the third case where $(n+q) > 3q$ or $n > 2q$, the eigenvalue-generalized eigenvector assignment is not possible. However adjustment of the number of system outputs may correct this situation, or the degrees of freedom in choosing the admissible generalized eigenvectors in Eq 18, could be utilized to assign a prescribed set of eigenvalues to the closed-loop system matrix A_c. To complete the presentation, the case where some eigenvalues are zeros will be treated next.

EIGENVALUE $\lambda_i=0$

Let λ_i be zero eigenvalue. In this situation Eqs 14 and 15 become

$$(I-BG_DC)^{-1}(A+BG_pC)U_1^{i,\mu,\rho} + (I-BG_DC)^{-1}BG_I U^{i,\mu,\rho} = U_1^{i,\mu,\rho-1}, \qquad \text{Eq 23}$$

$$CU_1^{i,\mu,\rho} = U^{i,\mu,\rho-1}. \qquad \text{Eq 24}$$

Let us assume without loss of generality that the output matrix C is expressed as $[I_q,0]$. Examination of Eq 24 reveals that q elements of $U_1^{i,\mu,\rho}$ are known in terms of $U^{i,\mu,\rho-1}$. The remaining (n-q) elements of $U_1^{i,\mu,\rho}$ along with $U^{i,\mu,\rho}$ are to be specified. Using Eqs 23 and 24 it can be shown that

$$[A,B]\begin{bmatrix} U_1^{i,\mu,\rho} \\ W^{i,\mu,\rho} \end{bmatrix} = U_1^{i,\mu,\rho-1} \qquad \text{Eq 25}$$

$$W^{i,\mu,\rho} = G_pU^{i,\mu,\rho-1}+G_IU^{i,\mu,\rho}+G_DU^{i,\mu,\rho-2}$$

where,

$$\rho = 1,2,\ldots,m_{i,\mu}, \qquad \mu = 1,2,\ldots,m_i. \qquad \text{Eq 26}$$

Using Eqs 25 and 26, we can express G_p, G_I and G_D in matrix form as

$$[G_p,\ G_I,\ G_D]\begin{bmatrix} \ldots\ldots,U^{i,\mu,\rho-1},\ldots\ldots \\ \ldots\ldots,U^{i,\mu,\rho}\ ,\ldots\ldots \\ \ldots\ldots,U^{i,\mu,\rho-2},\ldots\ldots \end{bmatrix} = \begin{bmatrix} \ldots\ldots,\ W^{i,\mu,\rho}\ ,\ldots\ldots \end{bmatrix} . \qquad \text{Eq 27}$$

Repeating this procedure for all the zero eigenvalues and following the procedure discussed previously for the non-zero eigenvalues, we can determine the output feedback gain matrices G_p, G_I and G_D via Eq 22.

SINGLE-INPUT SINGLE-OUTPUT SYSTEM WITH DISTINCT EIGENVALUES ($\neq 0$)

Consider a linear continuous single-input single-output system having the state variable form, Eq 1, and expressed in the canonical form

$$A=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}, \quad B=\begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix},$$

$$C=[1 \quad 0 \quad 0 \ldots\ldots\ldots\ldots 0],$$

where, in this case, m=q=1. Utilizing the results previously obtained for the non-zero eignvalue, it can be shown that the output feedback scalar gains g_p, g_i and g_d may be expressed as

$$[g_p, g_i, g_d]\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1/\lambda_1 & 1/\lambda_2 & \cdots & 1/\lambda_{n+1} \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n+1} \end{bmatrix} = \left[\sum_{j=0}^{n} a_j \lambda_1^{n-j}, \ldots\ldots, \sum_{j=0}^{n} a_j \lambda_{n+1}^{n-j} \right] \qquad \text{Eq 28}$$

From the last expression it is obvious that for a second order system the the output feedback scalar gains g_p, g_i and g_d are uniquely determined. These gains will assign any arbitrary set of distinct eigenvalues to the closed-loop matrix A_c. For higher order systems, n>2, the eigenvalue spectrum can not be completely assigned, which is noticed in the example reported in **(6)** .

NUMERICAL EXAMPLES

Several examples will be considered to illustrate the proposed technique.

EXAMPLE 1

Consider a third order system of the form, Eq 1, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

n=m=3 and q=2. Let us assume that we are interested in having a closed-loop eigenvalue spectrum of the following description: λ_1=-1 with geometric multiplicity $k_1 = 1$ and algebraic multiplicity $m_1 = 3$ and $\lambda_{2,3}$=-1 ± j with geometric multiplicity k_2=k_3=1 and algebraic multiplicity m_2=m_3=1. The design technique objective is to determine the output feedback gain matrices G_P, G_I and G_D such that the closed-loop system matrix A_c has the eigenvalue spectrum just described and the corresponding set of admissible self-conjugate generalized eigenvectors:

$$U^{1,1,1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \quad U^{1,1,2} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad U^{1,1,3} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -2 \end{bmatrix},$$

$$U^{2,1,1} = \begin{bmatrix} 1 \\ j \\ -j \\ -1/2-j/2 \\ 1/2-j/2 \end{bmatrix}, \quad U^{3,1,1} = \begin{bmatrix} 1 \\ -j \\ j \\ -1/2+j/2 \\ 1/2+j/2 \end{bmatrix}.$$

The eigenvalue spectrum does not contain zero eigenvalues, then we can apply the results reported for the non-zero eigenvalue. Since in this example n<2q, then we must impose some constraints on the elements of G_P, G_I and G_D. Three constraints are introduced giving results

i) the first column of G_D is set to zero and the corresponding G_P, G_I and G_D matrices are,

$$G_P = \begin{bmatrix} -3 & -1 \\ 0 & 0 \\ 1 & -4 \end{bmatrix}, \quad G_I = \begin{bmatrix} -3 & 1 \\ 3 & -1 \\ 8 & -4 \end{bmatrix}, \quad G_D = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & -6 \end{bmatrix},$$

ii) the row sum of G_P is set to zero and the corresponding G_P, G_I and G_D matrices are

$$G_P = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 4 & -4 \end{bmatrix}, \quad G_I = \begin{bmatrix} 1 & -1/3 \\ 3 & -1 \\ 11 & -5 \end{bmatrix}, \quad G_D = \begin{bmatrix} 4/3 & 0 \\ 0 & -1 \\ 1 & -6 \end{bmatrix},$$

iii) the row sum of G_D is set to zero and the corresponding G_P, G_I and G_D matices are:

$$G_P = \begin{bmatrix} -3 & -1 \\ 3 & 0 \\ 19 & -4 \end{bmatrix}, \quad G_I = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 26 & -10 \end{bmatrix}, \quad G_D = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 6 & -6 \end{bmatrix}.$$

Substituting the numerical values of G_P, G_I and G_D and the system A,B,C matrices into A_c, it can be shown for the three constraints that

$$A_c = \left[\begin{array}{ccc:cc} -3 & 0 & 0 & -3 & 1 \\ 0 & 0 & 1/2 & 3/2 & -1/2 \\ -1 & -3 & -2 & -4 & 0 \\ \hdashline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right].$$

It can be easily verified that A_c has the given set of eigenvalues and the corresponding set of self-conjugate generalized eigenvectors. This will complete the design procedure.

EXAMPLE 2

Consider the same system as given in Example 1, where the eigenvalue spectrum is changed to $\lambda_1=-1$ with geometric multiplicity $k_1=1$ and algebraic multiplicity $m_1=3$ and $\lambda_2=0$ with geometric multiplicity $k_2=1$ and algebraic multiplicity $m_2=2$. The set of admissible generalized eigenvectors to be assigned is given as:

$$U^{1,1,1}=\begin{bmatrix}1\\1\\0\\-1\\-1\end{bmatrix}, \quad U^{1,1,2}=\begin{bmatrix}-1\\0\\1\\0\\-1\end{bmatrix}, \quad U^{1,1,3}=\begin{bmatrix}1\\1\\-1\\-1\\-2\end{bmatrix},$$

$$U^{2,1,1}=\begin{bmatrix}0\\0\\1\\1\\-1\end{bmatrix}, \quad U^{2,1,2}=\begin{bmatrix}1\\-1\\1\\2\\1\end{bmatrix}.$$

Since the eigenvalue spectrum contains zero eigenvalue, then the results reported for zero eigenvalue are applicable. Note in this example that $CU^{2,1,1}=U^{2,1,0}=0$ and $CU^{2,1,2}=U^{2,1,1}$. Since $n<2q$, we impose some constraints on the elements of G_P, G_I and G_D. Three constraints are introduced and the results obtained are:

i) the first column of G_D is set to zero and the corresponding G_P, G_I and G_D matrices are

$$G_P=\begin{bmatrix}1 & 3\\-1 & -5\\-3 & -17\end{bmatrix}, \quad G_I=\begin{bmatrix}1 & 1\\-2 & -1\\-5 & -4\end{bmatrix}, \quad G_D=\begin{bmatrix}0 & 4\\0 & -2\\0 & -10\end{bmatrix},$$

ii) the row sum of G_P is set to zero and the correspnding G_P, G_I and G_D matrices are

$$G_P = \begin{bmatrix} 0.2 & -0.2 \\ 0.2 & -0.2 \\ 1 & -1 \end{bmatrix}, \quad G_I = \begin{bmatrix} 0.2 & 0.2 \\ -0.8 & 0.2 \\ -1 & 0 \end{bmatrix}, \quad G_D = \begin{bmatrix} 0.8 & 0.8 \\ -1.2 & 2.8 \\ -4 & 6 \end{bmatrix},$$

iii) the row sum of G_D is set to zero and the corresponding G_P, G_I and G_D matrices are

$$G_P = \begin{bmatrix} -1/3 & -7/3 \\ -1/3 & -7/3 \\ 1/2 & -11/3 \end{bmatrix}, \quad G_I = \begin{bmatrix} -1/3 & -1/3 \\ -4/3 & -1/3 \\ -5/3 & -2/3 \end{bmatrix}, \quad G_D = \begin{bmatrix} 4/3 & -4/3 \\ -2/3 & 2/3 \\ -10/3 & 10/3 \end{bmatrix}.$$

Substituting the numerical values of G_P, G_I and G_D and the system A,B,C matrices into A_c, it can be shown for the three constraints that

$$A_c = \left[\begin{array}{ccc:cc} -1/3 & -8/3 & 4/3 & -5/3 & -1/3 \\ -1/3 & -5/3 & 1/3 & -2/3 & -1/3 \\ 1 & -2 & -1 & 0 & -1 \\ \hdashline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right].$$

It can be verified that A_c has the given set of eigenvalues and the corresponding set of generalized eigenvectors.

EXAMPLE 3

Consider a voltmeter whose dynamics are governed by the state equation as given in Eq 1, where

$$A = \begin{bmatrix} 0 & 1 \\ -120 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0]$$

and $x_1(t)$ and $x_2(t)$ represent the deflection and the rate of change of the deflection as function of time. Since we are dealing with a single-input single-output system, the results obtained for single-input single-output are applicable. It is desirable to have closed-loop eigenvalues at $-6 \pm j12$ and -10 and the desired output $CX_d = x_{1d} = 1.0$. Applying Eq 28, the resulting scalar feedback gains are

$$[g_p, g_i, g_d] = [-180, -1800, -12].$$

Substituting these feedback gains in the closed-loop representation Eq 6, the voltmeter response in Fig. 2 case a is obtained. If only a proportional feedback with gain g_p is applied, then the voltmeter response to a step input in Fig 2, case b is obtained. The steady state error in this case is 0.4. This demonstrates the superiority of the PID controller over the usual P controller.

EXAMPLE 4

Consider a three-plate absorber controlled by inlet feed streams, where the transition from one equilibrium state to another is governed by a state equation as Eq 1 **(11)** . To illustrate the design technique proposed in this report, particular operating conditions are selected and the resulting system matrices are

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The objective is to determine the feedback gain matrices G_P, G_I and G_D such that the closed-loop matrix A_c has the eigenvalue spectrum: $\lambda_{1,2}=-5 \pm j$, $\lambda_{3,4}=-6 \pm j15$ and $\lambda_5=-21$ and the corresponding set of eigenvector subvectors

$$U_1^{1,1,1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad U_1^{2,1,1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad U_1^{3,1,1} = \begin{bmatrix} 0 \\ 1 \\ -4+j15 \end{bmatrix},$$

$$U_1^{4,1,1} = \begin{bmatrix} 0 \\ 1 \\ -4-j15 \end{bmatrix}, \quad U_1^{5,1,1} = \begin{bmatrix} 1 \\ 1 \\ -20 \end{bmatrix}.$$

The resulting feedback gain matrices where the first column of G_D is set to zero are

$$[G_P, G_I, G_D] = \left[\begin{array}{cc:cc:c} -8.0891 & -0.4930 & -131.5763 & -11.6254 & 0 \\ -0.0091 & 1.9867 & -1.1400 & -1.1924 & 0 \end{array} \begin{array}{c} -0.0404 \\ 0.9996 \end{array}\right]$$

CONCLUSIONS

In this paper, time-domain design techniques for PID controllers have been presented. The design technique utilizes system output. The resulting controller assigns a prescribed set of self-conjugate eigenvalues and a corresponding set of admissible self-conjugate eigenvectors to the closed-loop system matrix. The proposed techniques are flexible, straightforward and computationally simpler over known techniques since they do not impose any structural constraints on the PID controller feedback gain matrices of unity rank **(3,7,9)**. Moreover, the computation of the open-loop transfer function matrix is avoided **(7)**. The flexibility in assigning the generalized eigenvectors could be utilized in minimizing eigenvalue sensitivities, quadratic performance index or any other performance criterion.

The eigenstructure approach adopted in this paper is similar to the eigenstructure approach utilized in the eigenvalue-generalized eigenvector assignment using state feedback (12,13) , except that it is extended to the case of PID controller.

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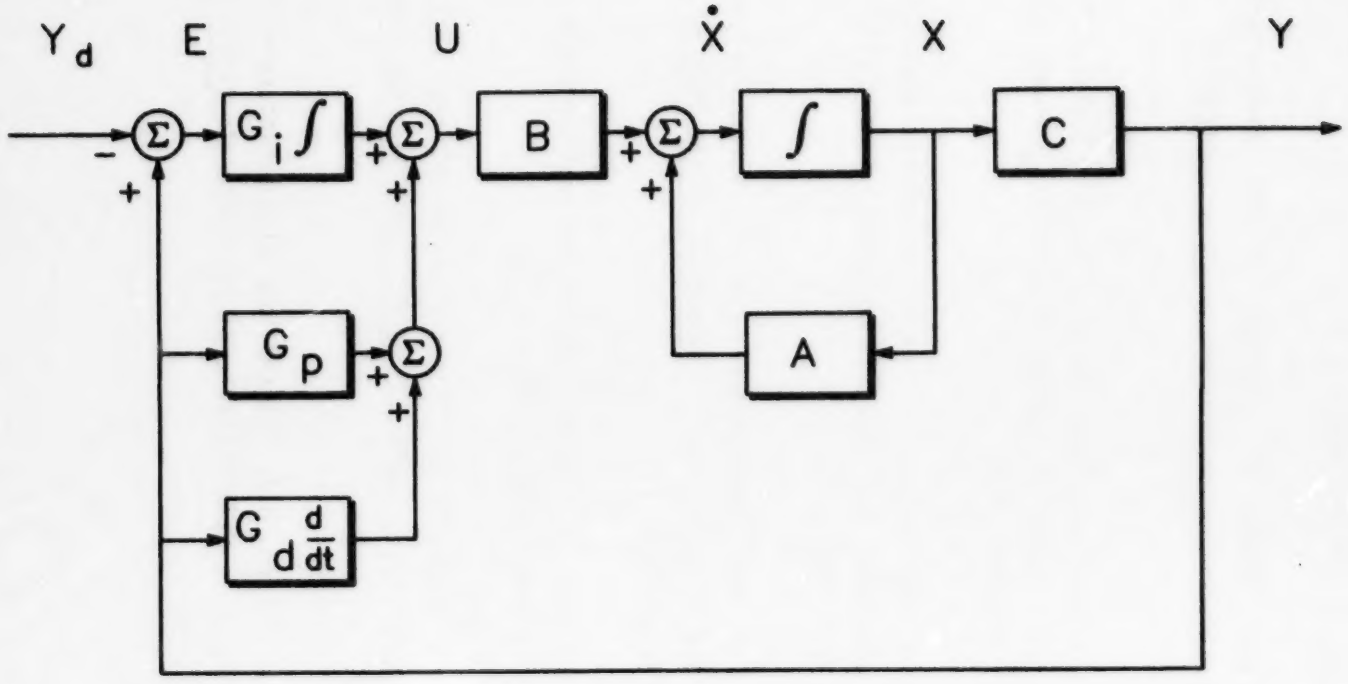

Fig. 1 - A linear continuous multivariable system with PID controller

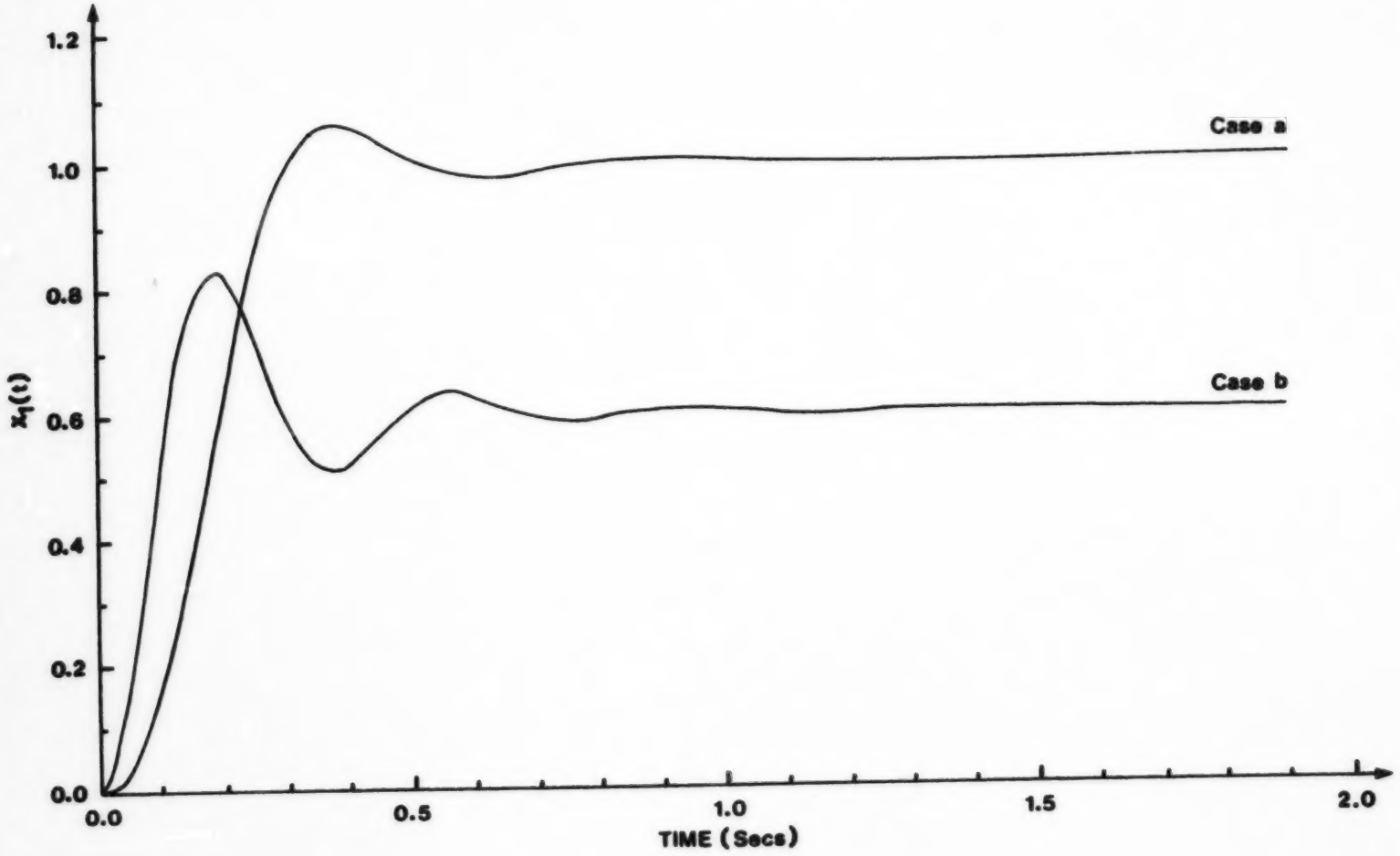

Fig. 2 - A voltmeter time response with PID controller (case a) and P controller (case b)